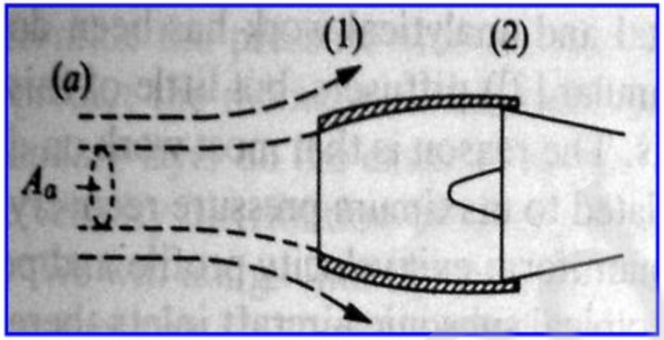
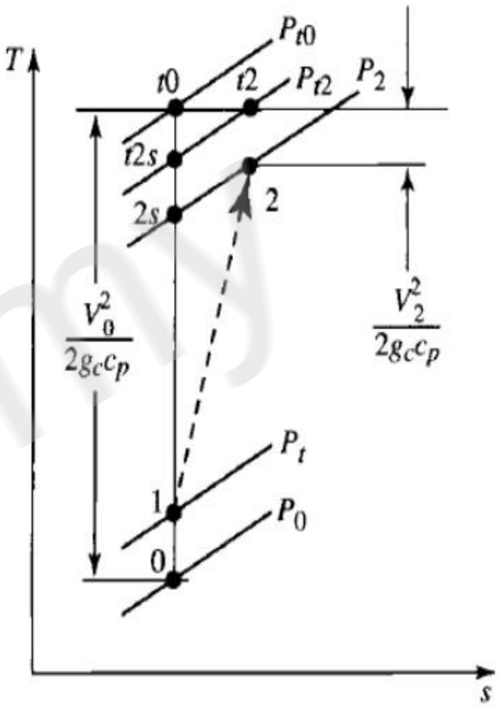
Propulsion

Diffuser isentropic efficiency





$$\text{Diffuser isentropic efficiency} = \frac{\text{Isentropic enthalpy rise}}{\text{Actual enthalpy rise}}$$

$$\eta_d = \frac{h_{t2s} - h_0}{h_{t2} - h_0} = \frac{h_{t2s} - h_0}{h_{t0} - h_0} = \frac{T_{t2s} - T_0}{T_{t2} - T_0}$$

We know that $\pi_d = \frac{P_{t2}}{P_{t0}}, \tau_d = \frac{T_{t2}}{T_{t0}}, \tau_r = \frac{T_{t2}}{T_0}$

$$\eta_d = \frac{\left(\pi_d \pi_r\right)^{\frac{\gamma-1}{\gamma}} - 1}{\tau_r - 1}$$

Compressor isentropic efficiency

$$\text{Compressor isentropic efficiency} = \frac{\text{Isentropic work for a given } \pi_c}{\text{Actual work for a given } \pi_c}$$

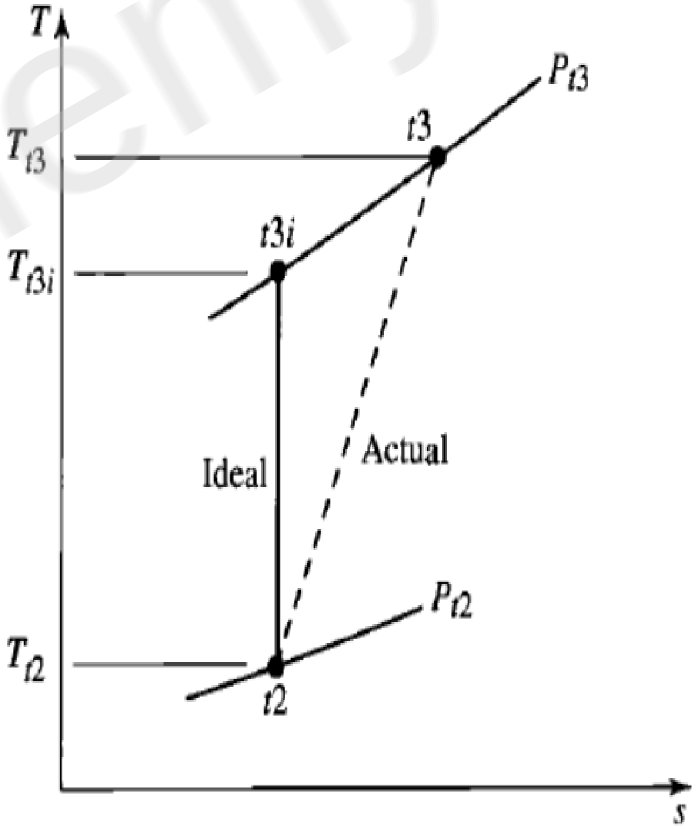
$$\eta_c = \frac{h_{t3i}-h_{t2}}{h_{t3}-h_{t2}} = \frac{T_{t3i}-T_{t2}}{T_{t3}-T_{t2}} = \frac{\left(\dfrac{T_{t3i}}{T_{t2}}-1\right)}{\left(\dfrac{T_{t3}}{T_{t2}}-1\right)}$$

$$\frac{T_{t3}}{T_{t2}}-1=\tau_c-1 \quad \text{and} \quad \frac{T_{t3i}}{T_{t2}}=\left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}}=(\pi_c)^{\frac{\gamma-1}{\gamma}}$$

$$\eta_c = \frac{(\pi_c)^{\frac{\gamma-1}{\gamma}}-1}{\tau_c-1}$$

Also if we know π_c and η_c, then

$$T_{t3} = \left[1+\frac{1}{\eta_c}\left(\pi_c^{\frac{\gamma-1}{\gamma}}-1\right)\right]$$



Combustion efficiency

$$\text{Combustion efficiency, } \eta_b = \frac{\text{Heat energy released}}{\text{Heat energy available (with fuel)}}$$

$$\eta_b = \frac{(\dot{m}_i + \dot{m}_{fuel})C_{p4}T_{t4} - \dot{m}_i C_{p3}T_{t3}}{\dot{m}_{fuel} Q_R}$$

Nozzle isentropic efficiency

$$\text{Nozzle isentropic efficiency} = \frac{\text{Actual enthalpy drop}}{\text{Ideal enthalpy drop}}$$

$$\eta_n = \frac{h_{t5} - h_9}{h_{t5} - h_{9i}} = \frac{T_{t5} - T_9}{T_{t5} - T_{9i}}$$

Turbine isentropic efficiency

$$\text{Turbine isentropic efficiency} = \frac{\text{Actual work for a given } \pi_t}{\text{Ideal work for a given } \pi_t}$$

$$\eta_t = \frac{h_{t4} - h_{t5}}{h_{t4} - h_{t5i}} = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5i}} = \frac{\left(1 - \frac{T_{t5}}{T_{t4}}\right)}{\left(1 - \frac{T_{t5i}}{T_{t4}}\right)}$$

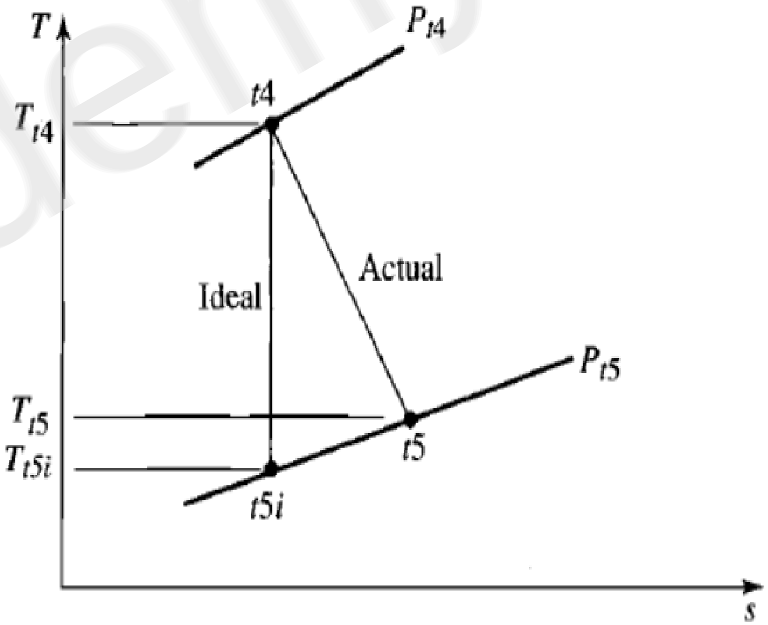
$$1 - \frac{T_{t5}}{T_{t4}} = 1 - \tau_t - 1 \quad \text{and} \quad \frac{T_{t5i}}{T_{t4}} = \left(\frac{P_{t5}}{P_{t4}}\right)^{\frac{\gamma-1}{\gamma}} = (\pi_t)^{\frac{\gamma-1}{\gamma}}$$

$$\eta_t = \frac{1 - \tau_t}{1 - (\pi_t)^{\frac{\gamma-1}{\gamma}}}$$

Also if we know π_t and η_t, then

$$T_{t3} = T_{t4}\left[1 - \eta_t\left(1 - \pi_t^{\frac{\gamma-1}{\gamma}}\right)\right]$$

$$P_{t5} = P_{t4}\left[\frac{1}{\eta_t}\left(1 - \frac{T_{t5}}{T_{t4}}\right)\right]^{\frac{\gamma}{\gamma-1}}$$



Typical values of isentropic efficiencies are:

$\eta_d \approx 0.97$

$\eta_c \approx 0.85$

$\eta_t \approx 0.90$

$\eta_n \approx 0.98$

Polytropic efficiency

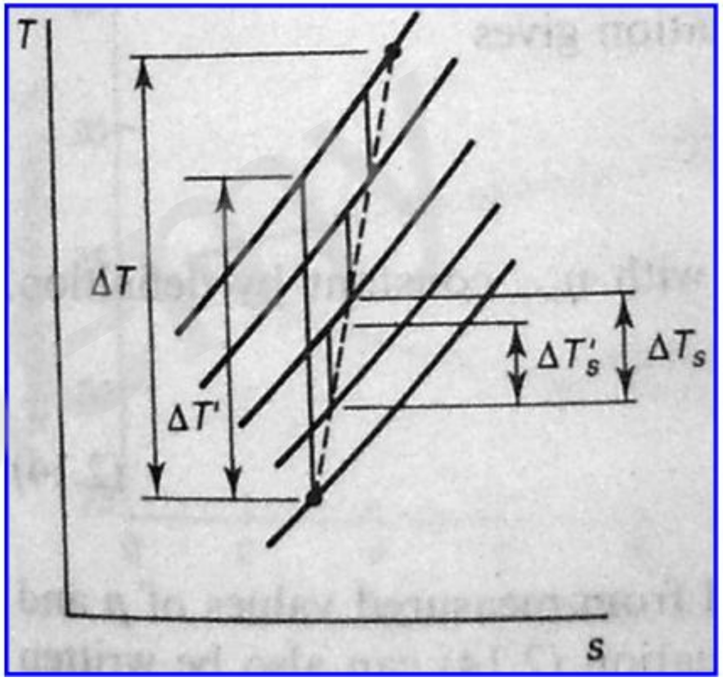
- It is defined as the isentropic efficiency of *an infinitesimal stage* in the compression process.

$$\text{Polytropic efficiency}, \eta_{poly} = \frac{\text{Ideal work done for an infinitesimal step in the compression process}}{\text{Actual work done for an infinitesimal step in the compression process}}$$

- It is constant throughout the entire compression process.

From definition, $\eta_{poly} = \dfrac{dT_t^{'}}{dT_t}$

For an isentropic process, $\dfrac{T_t}{p_t^{\frac{\gamma-1}{\gamma}}} = \text{constant}$



In differential form, $\dfrac{dT_t^{'}}{T_t} = \dfrac{\gamma-1}{\gamma}\dfrac{dp_t}{p_t}$

$$\frac{\eta_{poly} dT_t}{T_t} = \frac{\gamma-1}{\gamma}\frac{dp_t}{p_t}$$

$$\eta_{poly}\frac{dT_t}{T_t} = \frac{\gamma-1}{\gamma}\frac{dp_t}{p_t}$$

Integrate between the compressor inlet and the outlet

$$\int_1^2 \eta_{poly}\frac{dT_t}{T_t} = \int_1^2 \frac{\gamma-1}{\gamma}\frac{dp_t}{p_t}$$

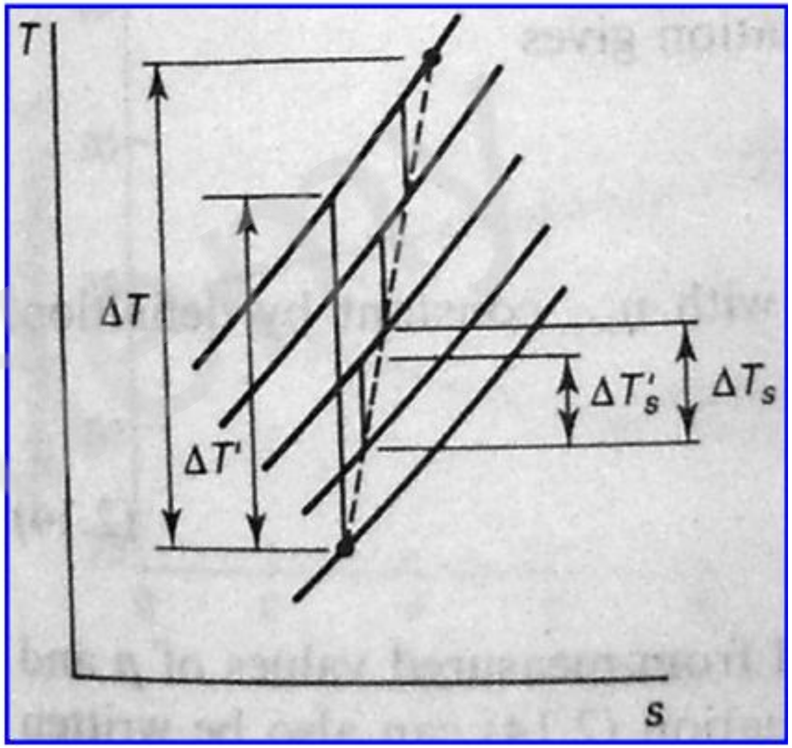
From definition, η_{poly} is constant and γ is constant

$$\eta_{poly}\ln\left(\frac{T_{t2}}{T_{t1}}\right)=\frac{\gamma-1}{\gamma}\ln\left(\frac{p_{t2}}{p_{t1}}\right)$$

$$\eta_{poly}=\frac{\frac{\gamma-1}{\gamma}\ln\left(\frac{p_{t2}}{p_{t1}}\right)}{\ln\left(\frac{T_{t2}}{T_{t1}}\right)}$$

$$\frac{T_{t2}}{T_{t1}}=\left(\frac{p_{t2}}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma\eta_{poly}}}$$

$$\frac{T_{t2}}{T_{t1}}=\left(\frac{p_{t2}}{p_{t1}}\right)^{\frac{n-1}{n}}\text{, where }\frac{n-1}{n}=\frac{\gamma-1}{\gamma\eta_{poly}}$$



Relationship between η_c & η_{poly}

$$\eta_c = \frac{T'_{t3} - T_{t2}}{T_{t3} - T_{t2}} = \frac{\left(\frac{T'_{t3}}{T_{t2}}\right) - 1}{\left(\frac{T_{t3}}{T_{t2}}\right) - 1} = \frac{\left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_{t2}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma\eta_{poly}}} - 1}$$

- Similar relationship can be obtained for turbine also.