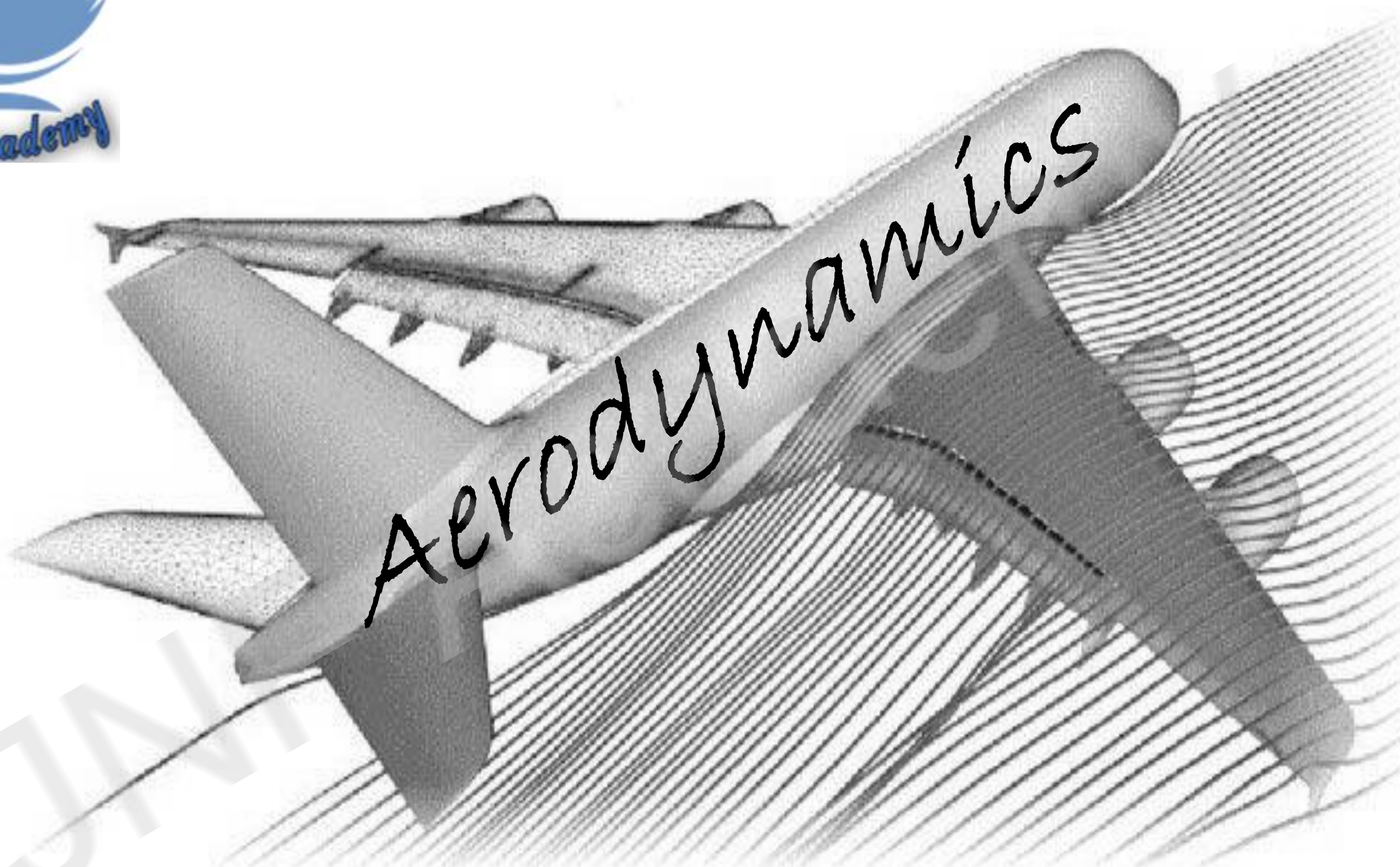

Aerodynamics

Critical Mach Definition

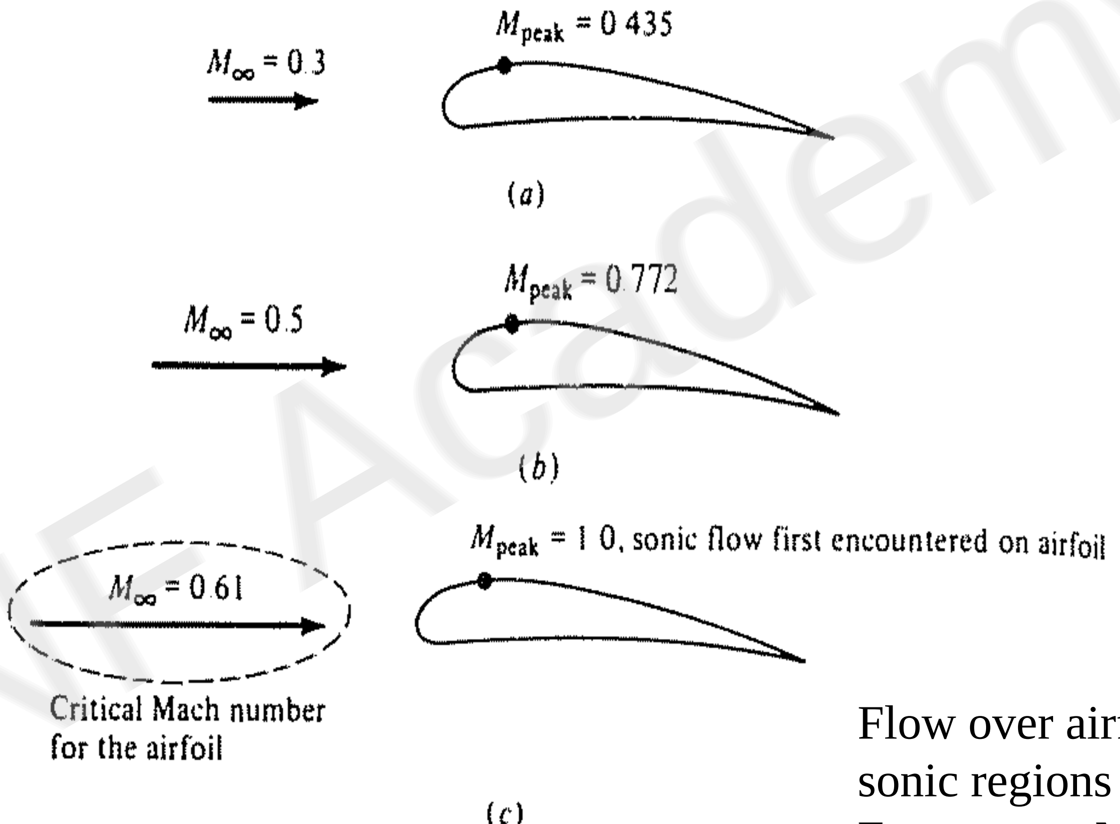
- In aerodynamics, the critical Mach Number (M_{cr} or M_{crit}) of an aircraft is the lowest Mach number at which the airflow over any part of the aircraft reaches the speed of sound.

- For all aircraft in flight, the speed of the airflow around the aircraft is not exactly the same as the airspeed of the aircraft due to the airflow speeding up and slowing down to travel around the aircraft structure.

- At the Critical Mach number, local airflow near some areas of the airframe reaches the speed of sound, even though the aircraft itself has an airspeed lower than Mach 1.0. This creates a weak shock wave. In aircraft not designed for transonic or supersonic flight, speeds greater than the Critical Mach number will cause the drag coefficient to increase suddenly causing a dramatic increase in total drag and changes to the airflow over the flight control surfaces will lead to deterioration in control of the aircraft.

CRITICAL MACH NUMBER, M_{CR} (5.9)

- As air expands around top surface near leading edge, velocity and M will increase
- Local $M > M_\infty$



Flow over airfoil may have sonic regions even though Free stream $M_\infty < 1$

INCREASED DRAG

Parameters affecting the critical Mach no:

Thick airfoils have a lower critical Mach number than thin airfoils

Desirable to have MCR as high as possible

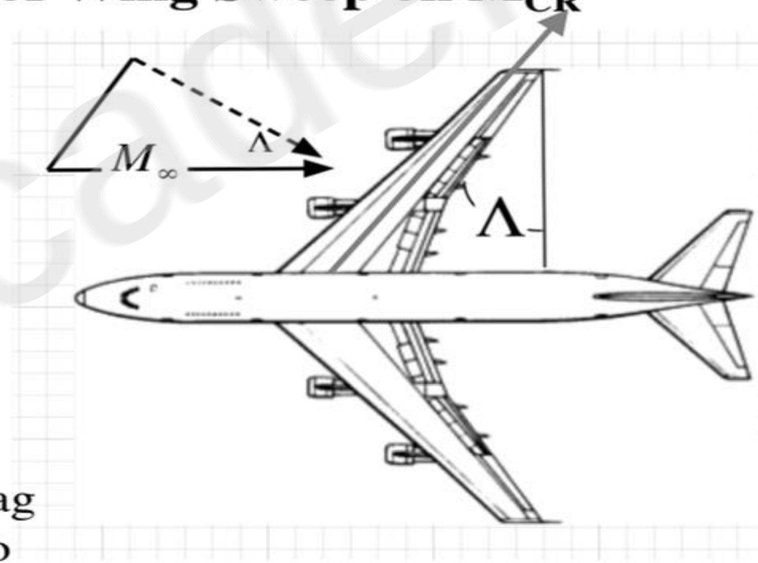
Implication for design → high speed wings usually design with thin airfoils

Effect of Wing Sweep on M_{CR}

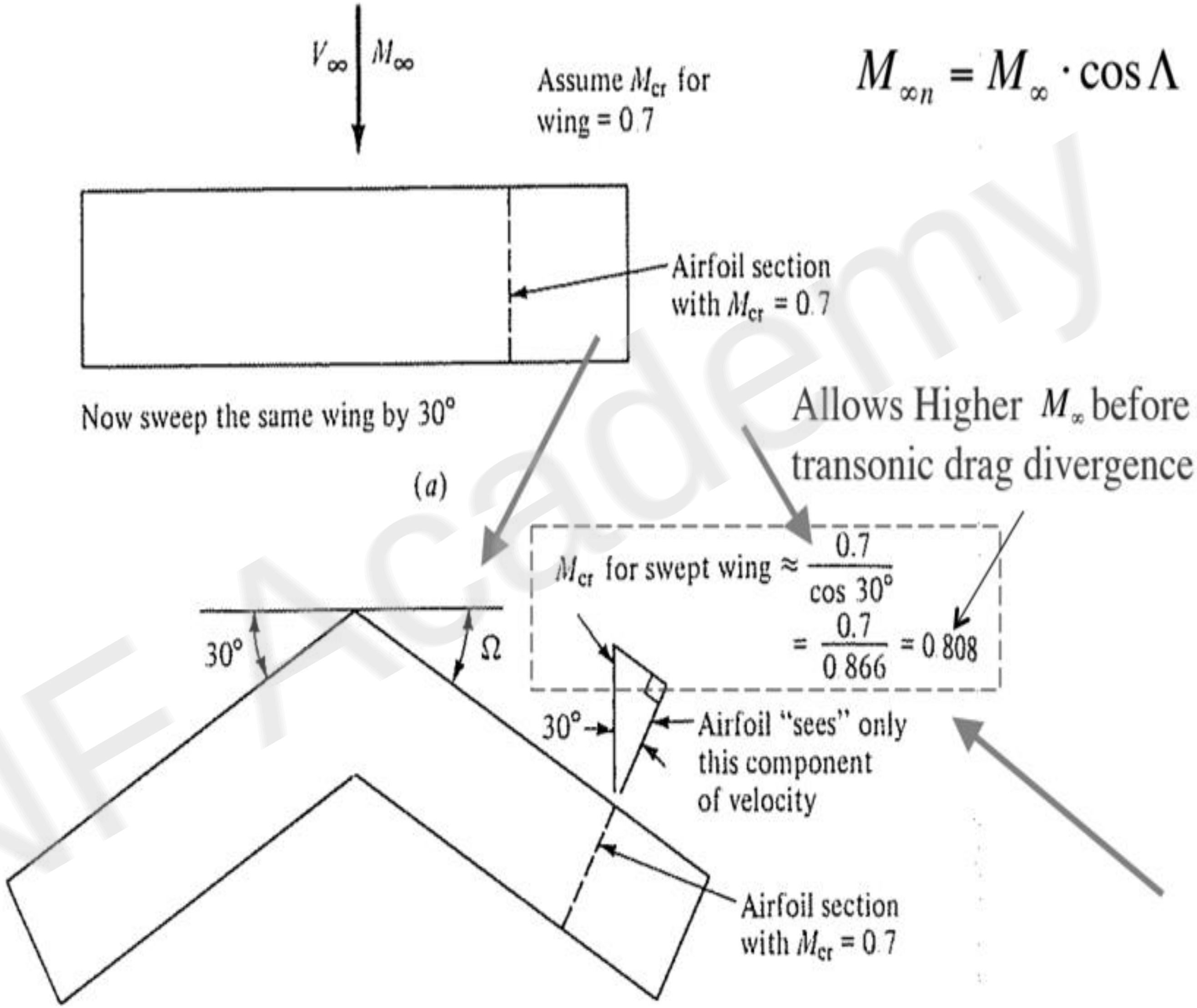
$$M_{\infty n} = M_{\infty} \cdot \cos\Lambda$$

- Wing "sees" only Component of Mach number normal to leading edge

- By sweeping wings of <u>subsonic</u> aircraft, drag divergence is delayed to higher Mach numbers



$$M_{\infty n} = M_{\infty} \cdot \cos\Lambda$$



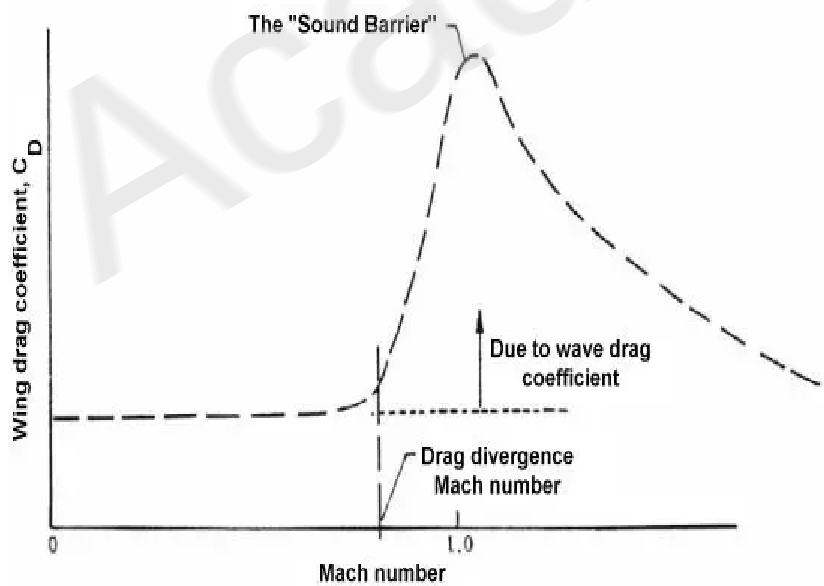
Drag Divergence Mach Definition

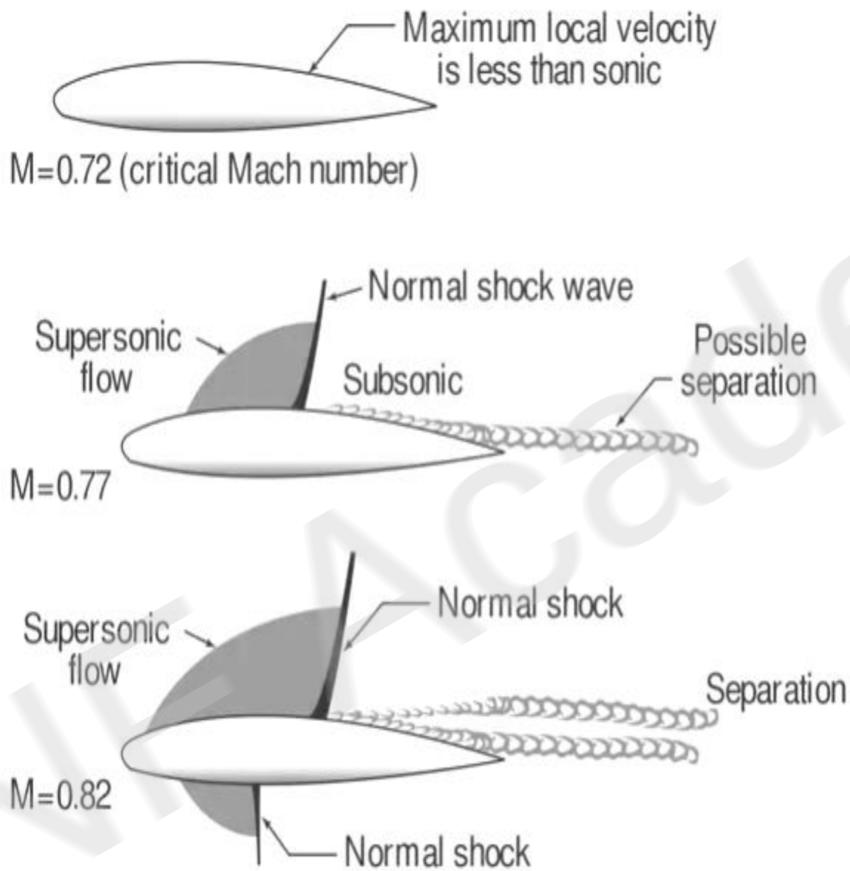
The **drag-divergence Mach number** is the Mach no at which the drag on an airfoil or airframe begins to increase rapidly as the Mach number continues to increase. This increase can cause the drag coefficient to rise to more than ten times its low speed value.
The value of the drag-divergence Mach number is typically greater than 0.6; therefore it is a transonic effect. The drag-divergence Mach number is usually close to, and always greater than, the Critical number. Generally, the drag coefficient peaks at Mach 1.0 and begins to decrease again after the transition into the supersonic regime above approximately Mach 1.2.
The large increase in drag is caused by the formation of a shock wave on the upper surface of the airfoil, which can induce flow separation and adverse pressure gradients on the aft portion of the wing. This effect requires that aircraft intended to fly at supersonic speeds have a large amount of thrust. In early development of transonic and supersonic aircraft, a steep dive was often used to provide extra acceleration through the high-drag region around Mach 1.0.
This steep increase in drag gave rise to the popular false notion of an unbreakable sound barrier, because it seemed that no aircraft technology in the foreseeable future would have enough propulsive force or control authority to overcome it. Indeed, one of the popular analytical methods for calculating drag at high speeds, the Prandtl–Glauert rule, predicts an infinite amount of drag at Mach 1.0.

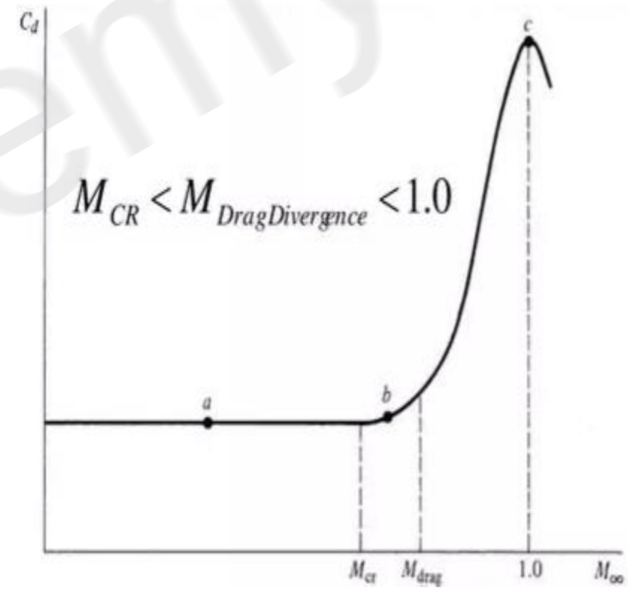
Drag Divergence Mach - Significance

➢ It takes much more thrust to push the airplane faster as you enter the M-d zone, because of the greatly increased wing drag.

➢ The normal shock wave being formed at the top of the wing will cause buffeting on the aft section of the wing - the structure of the wing must be strong enough to withstand these buffets, and the controls must be able to function in spite of the presence of the buffeting. These two requirements demand considerable attention during design.




Maximum local velocity is less than sonic
M=0.72 (critical Mach number)
Normal shock wave
Supersonic flow
Subsonic
Possible separation
M=0.77
Normal shock
Supersonic flow
Separation
M=0.82
Normal shock


C_d
$M_{CR} < M_{DragDivergence} < 1.0$
a
b
c
M_{cr}
M_{drag}
1.0
M_∞

For inviscid, compressible flow past a thin airfoil, shown in the figure, free-stream Mach number and pressure are denoted by M_∞ and p_∞ respectively. Ratio of pressure at point **A** and p_∞ is 0.8 and specific heat ratio is 1.4. If the Mach number at point **A** is 1.0 and rest of the flow field is subsonic, the value of M_∞ is

(A) 2.95 (B) 0.79 ✓ (C) 1.18 (D) 0.64

Soln

Gn. $\frac{P_A}{P_\infty} = 0.8$, $M_A = 1$, $\gamma = 1.4$, flow is compressible, $M_\infty = ?$

Flow over an airfoil is isentropic, hence Total pressure will be constant throughout the flow. $\Rightarrow P_{0\infty} = P_{0A}$

From isentropic relations,

$$\frac{P_{0A}}{P_A} = \left(1 + \frac{\gamma-1}{2} M_A^2\right)^{\gamma/\gamma-1} \text{ --- (1)}$$

$$\frac{P_{0\infty}}{P_\infty} = \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/\gamma-1} \text{ --- (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{P_A}{P_\infty} = \frac{(1+0.2M_\infty^2)^{3.5}}{(1+0.2(1)^2)^{3.5}} = 0.8$$

$$\boxed{M_\infty = 0.79}$$

With increase in airfoil thickness, the critical Mach number for an airfoil is likely to

(A) decrease. (B) increase. (C) remain unchanged. (D) be undefined.

Soln

As per the notes, as airfoil thickness increases critical Mach number decreases.

The critical Mach number for a flat plate of zero thickness, at zero angle of attack, is 1

Soln.

By the defn. of critical Mach number, given in the notes

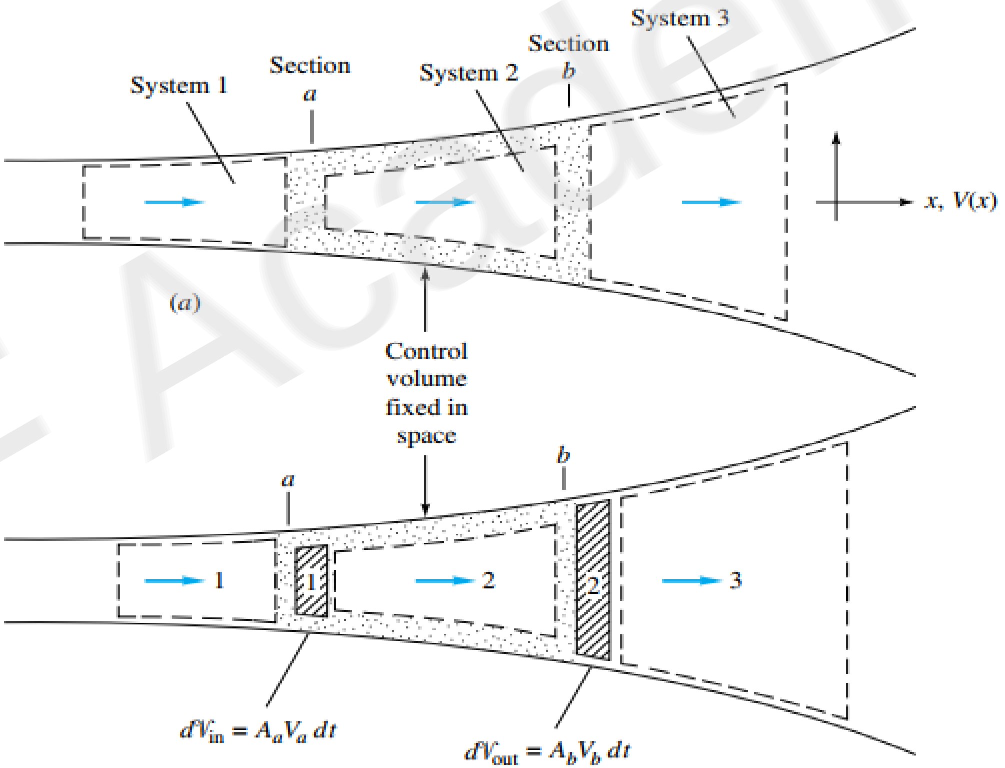
The free Stream Mach number in which the sonic Mach number will be attained in the flow.

For a flat plate with zero thickness and zero angle of attack the flow will not get accelerated, so the critical Mach number is same as the free stream Mach number, when $M_\infty = 1$.

Law of Conservation of Mass

Mass can neither be created nor be destroyed. Mass = constant. DM/DT = 0.

For fluids, Control Volume approach is required. – Reynolds Transport Theorem.



1. Rate of change of B within the Control Volume

2. The flux of B passing out of control surface

3. The flux of B passing into the control surface

$$B_{cv} = \int_{cv} \beta \rho \, dV \quad , \quad \beta = \frac{dB}{dm}$$

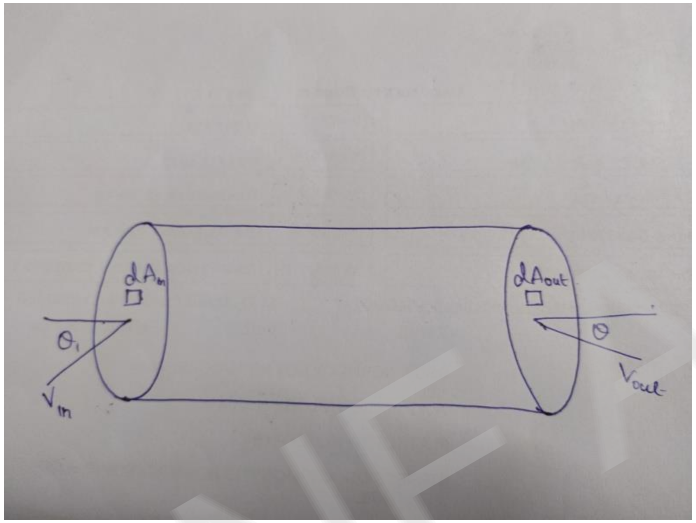
$$\frac{d}{dt}(B_{cv}) = \frac{1}{dt} B_{cv}(t+dt) - \frac{1}{dt} B_{cv}(t)$$

$$= \frac{1}{dt}\left[B_2(t+dt) - (\beta \rho \, dV)_{out} + (\beta \rho \, dV)_{in}\right] - \frac{1}{dt}\left[B_2(t)\right]$$

$$= \frac{1}{dt}\left[B_2(t+dt) - B_2(t)\right] - (\beta \rho A V)_{out} + (\beta \rho A V)_{in}$$

$$\frac{d}{dt}(B_{syst}) = \underbrace{\frac{d}{dt}\left(\int_{cv} \beta \rho \, dV\right)}_{(1)} + \overbrace{(\beta \rho A V)_{out}}^{(2)} - \underbrace{(\beta \rho A V)_{in}}_{(3)}$$

Generalizing the RTT to any arbitrary control Volume



$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{CV} \beta \rho \, dV\right) + \int_{CS} \beta \rho V \cos\theta \, dA_{out} - \int_{CS} \beta \rho V \cos\theta \, dA_{in}$$

$$\text{Flux terms} = \int_{CS} \beta \rho V_n \, dA_{out} - \int_{CS} \beta \rho V_n \, dA_{in}$$

$$= \int_{CS} \beta \, d\dot{m}_{out} - \int_{CS} \beta \, d\dot{m}_{in}$$

Reynolds Transport theorem to Mass Conservation.

Lets assume the property B to be mass and substitute M = m

$$\left(\frac{dm}{dt}\right)_{syst} = \int_{CV} \frac{\partial \rho}{\partial t}\, dV + \int_{CS} \rho (V \cdot n)\, dA = 0$$

From Gauss Divergence theorem

a) $$\iint_S \bar{F} \cdot ds = \iiint_V \nabla \cdot \bar{F}\, dv$$

b) $$\iint_S \phi \cdot ds = \iiint_V (\nabla \phi)\, dv$$

$$\int_{CV} \left[\frac{d\rho}{dt} + \nabla \cdot (\rho \bar{V})\right] dV = 0$$

$$\frac{d\rho}{dt} + \nabla \cdot (\rho \bar{V}) = 0$$

For Incompressible and steady flow.

$$\frac{d\rho}{dt} = 0$$

$$\nabla \cdot (\rho \bar{V}) = 0$$

$$\nabla \cdot \bar{V} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Problem

GATE 2013

Consider a compressible flow where an elemental volume of the fluid is $\delta\vartheta$, moving with velocity $\vec{V}$. Which one of the following expressions is TRUE?

(A) $\nabla \cdot \vec{V} = \frac{1}{\delta\vartheta}\frac{D\delta\vartheta}{Dt}$

(B) $\nabla \cdot (\nabla \times \vec{V}) = \frac{1}{\delta\vartheta}\frac{D\delta\vartheta}{Dt}$

(C) $\nabla \cdot \frac{D\vec{V}}{Dt} = \frac{1}{\delta\vartheta}\frac{D\delta\vartheta}{Dt}$

(D) $\vec{V} \cdot (\nabla \times \vec{V}) = \frac{1}{\delta\vartheta}\frac{D\delta\vartheta}{Dt}$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho V = 0$$

$$\rho = \frac{1}{\delta V}$$

$$\nabla \cdot V = -\frac{1}{\rho}\frac{\partial \rho}{\partial t}$$

$$\nabla \cdot V = -\delta V \frac{\partial}{\partial t}(1/\delta V)$$

$$\nabla \cdot V = -\delta V \frac{-1}{(\delta V)^2}\frac{\partial(\delta V)}{\partial t}$$

$$\nabla \cdot V = \frac{1}{\delta V} \cdot \frac{\partial(\delta V)}{\partial t}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho V = 0$$

$$\rho = \frac{1}{\delta V}$$

$$\nabla \cdot V = -\frac{1}{\rho} \frac{\partial \rho}{\partial t}$$

$$\nabla \cdot V = -\delta V \frac{\partial}{\partial t}(1/\delta V)$$

$$\nabla \cdot V = -\delta V \frac{-1}{(\delta V)^2} \frac{\partial(\delta V)}{\partial t}$$

$$\nabla \cdot V = \frac{1}{\delta V} \cdot \frac{\partial(\delta V)}{\partial t}.$$