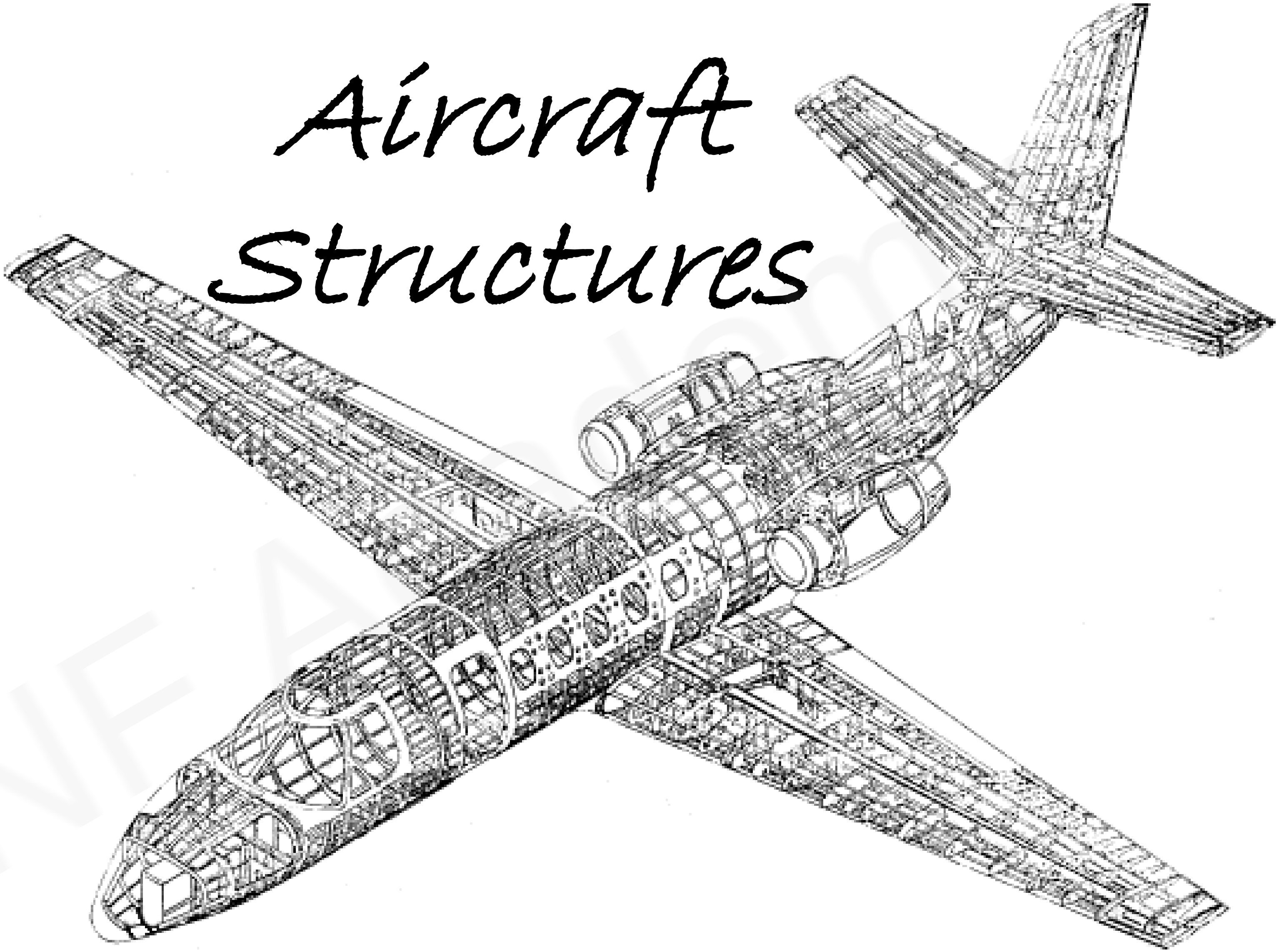
Aircraft Structures



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Castigliano's Theorem

I^{st} theorem :-

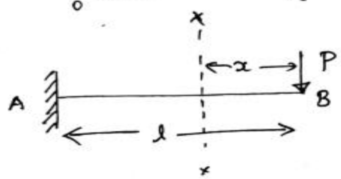
$$P = \frac{\partial U}{\partial \delta}$$

III^{rd} theorem :-

$$\delta = \frac{\partial U}{\partial P}$$

$$U = \int_0^l \frac{M^2}{2EI}\,dx \quad \text{(strain energy due to bending)}$$

Problem - 1



EI - constant

Find deflection at tip using castigliano's theorem

$$\delta = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial P}\,dx$$

$-ve$ (Hogging)

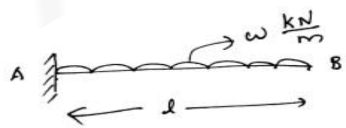
$$M_x = -Px$$

$$\frac{\partial M}{\partial P} = -x$$

$$EI\delta = \int_0^l (-Px)(-x)\,dx = P\left[\frac{x^3}{3}\right]_0^l$$

$$\boxed{\delta_B = \frac{Pl^3}{3EI}}$$

Problem - 2



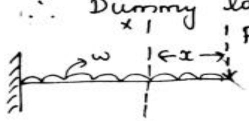
EI - const

Use Dummy load method

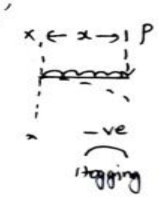
$\delta_B = ?$

To find the deflection at point B, we need a point load at point 'B'

$\therefore$ Dummy load 'P' is applied at the tip 'B'



$$M_x = -Px - wx\cdot\frac{x}{2}$$



8

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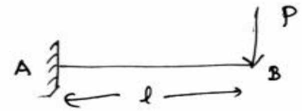
$$\frac{\partial M}{\partial P} = -x$$

$$\delta_B = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial P}\,dx = \frac{1}{EI}\int_0^l \left(-Px - \frac{wx^2}{2}\right)(-x)\,dx$$

$$EI\delta_B = \cancel{P}\left[\frac{x^3}{3}\right]_0^l + \frac{w}{2}\left[\frac{x^4}{4}\right]_0^l \quad \because P = 0 \text{ (Dummy load)}$$

$$\boxed{\delta_B = \frac{wl^4}{8EI}}$$

Problem − 3

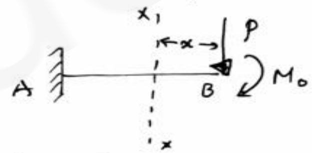


EI - const

Find slope at 'B'

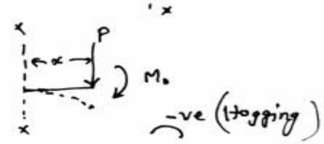
To find slope at 'B', we need a moment (couple) at the point 'B'

∴ Dummy moment M_0 is applied at tip



$$\theta_B = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx$$

$$M_x = -Px - M_0$$



$$\frac{\partial M}{\partial M_0} = -1$$

$$EI\theta_B = \int_0^l (-Px - M_0)(-1)\,dx$$

$$= \int_0^l Px\,dx + \int_0^l \cancel{M_0}\,dx$$

(Dummy moment $M_0 = 0$)

$$= P\left[\frac{x^2}{2}\right]_0^l$$

$$\boxed{\theta_B = \frac{Pl^2}{2EI}}$$

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Unit load method

$$\boxed{\delta = \int_0^l \frac{Mm}{EI}\,dx}$$

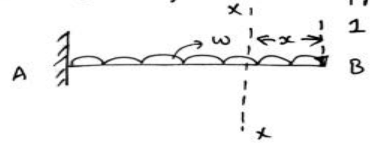
EI – const

Problem – 2

Find δ_B using unit load method.

To find deflection at 'B', we need a point load at the point 'B'

$\therefore$ unit load $P = 1$ is applied at the tip 'B'



$$\cancel{M_x = (-1 \times x) - \left(w \cdot x \cdot \frac{x}{2}\right) = -x - \frac{wx^2}{2}}$$

$$\delta_B = \int_0^l \frac{Mm}{EI}\,dx$$

$M \rightarrow$ moment due to external load

$m \rightarrow$ " " unit load

$$M = -\frac{wx^2}{2} \quad ; \quad m = -1 \cdot x$$

$$EI\,\delta_B = \int_0^l \left(-\frac{wx^2}{2}\right)(-x)\,dx$$

$$= \frac{w}{2}\left[\frac{x^4}{4}\right]_0^l$$

$$\boxed{\delta_B = \frac{wl^4}{8EI}}$$

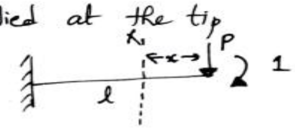
Problem – 3

Find the slope at 'B' using unit load method.

To find slope at 'B', we need a moment at the tip 'B'

$\therefore$ Unit moment $m = 1$ is applied at the tip

$$\theta_B = \int_0^l \frac{Mm}{EI}\,dx$$



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$$M_x = -Px$$

$$m = -1$$

$$\theta_B = \int_0^{l} \frac{(-Px)(-1)}{EI}\, dx$$

$$= \frac{P}{EI}\left[\frac{x^2}{2}\right]_0^{l}$$

$$\boxed{\theta_B = \frac{Pl^2}{2EI}}$$

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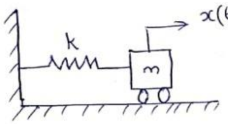
Methods of writing equation of motion:

SDOF - system (Idealized SDOF - mathematical model)

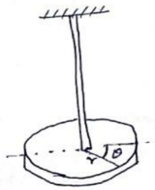
①



Spring - mass system

(x(t) - Translational displacement)

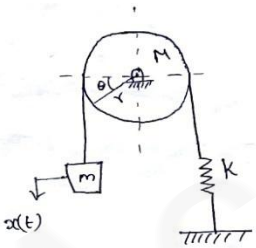
②



Torsional pendulum

(θ - rotational displacement)

③



M – mass of the pulley

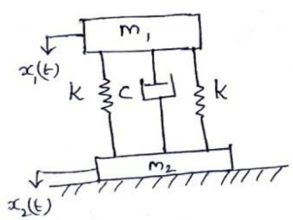
m - suspended mass

$x = r\sin\theta$ (Translational displacement depends on rotational displacement - so it is SDOF system)

θ is small

$\boxed{x = r\theta}$

④



Relative displacement

$x(t) = x_1(t) - x_2(t)$

Disp of mass m_1 w.r.t m_2

Displacement of mass at any given time, can be obtained from Eqn of motion.

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Methods for eqn of motion:

1. Simple Harmonic motion (SHM) Principle
2. Newton's law of motion
3. Energy method
4. Rayleigh's method
5. D'Alembert's principle

① SHM - characteristics

* Acceleration will be always <u>proportional</u> to the distance (or) displacement of the body measured along the path
* Body always directed towards the equilibrium position. Hence this direction is <u>opposite</u> to its motion.

$$\therefore \quad \ddot{x} \propto (-x)$$

Eqn of motion $\ddot{x} = -\omega x$

$$\boxed{\ddot{x} + \omega x = 0} \quad (2^{nd} \text{ order ODE})$$

$x \rightarrow$ disp $\qquad \dfrac{dx}{dt} = \dot{x}$ = Velocity

$$\frac{d^2x}{dt^2} = \frac{d\dot{x}}{dt} = \ddot{x} \rightarrow \text{acceleration}$$

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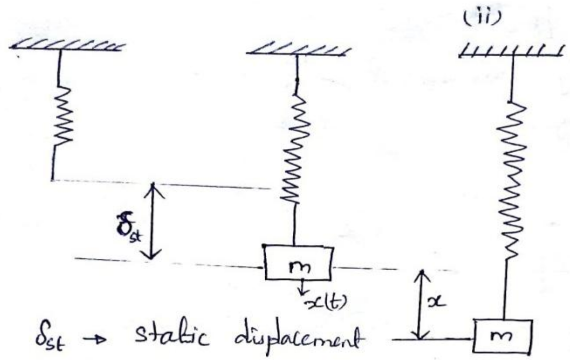
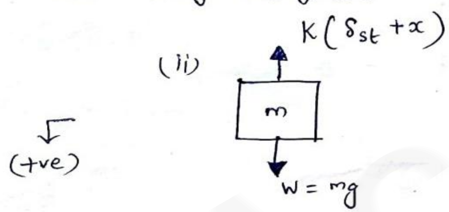
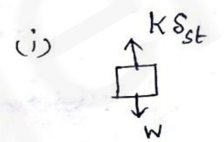
② Newton's Law :



$\delta_{st} \rightarrow$ static displacement

Free Body Diagram





$W = K\delta_{st}$

$$\text{Net Force } (F) = Ma$$

$$+W - K(\delta_{st} + x) = m\ddot{x}$$

$$+K\cancel{\delta_{st}} - K\cancel{\delta_{st}} - Kx = m\ddot{x}$$

$$\boxed{m\ddot{x} + Kx = 0}$$

- Represent all the forces vectorially (FBD)
- Sum all the forces (along ΣH, ΣV - or desired direction)
- Then write eqn of motion
- This method will be suitable only to identify the response of the system.

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3) Energy method :

- It can be applied only to conservative system.
- Kinetic Energy component (function of velocity)
- Potential Energy " (func of displacement)

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{x}^2$$

$$P.E = \frac{1}{2} k x^2$$

Total Energy = K.E + P.E = const ($\because$ conservative system)

$$\therefore \frac{d(T.E)}{dt} = 0$$

$$\frac{1}{\cancel{2}} m(\cancel{2}\dot{x})\ddot{x} + \frac{1}{\cancel{2}} k(\cancel{2}x)\dot{x} = 0$$

$$\dot{x}(m\ddot{x} + kx) = 0$$

$\dot{x}$ - velocity $\neq 0$

$$\therefore \boxed{m\ddot{x} + kx = 0}$$

This method works on energy principle

$$T.E \begin{cases} K.E \\ P.E \\ D.E \text{ (Dissipated energy)} \\ \vdots \end{cases}$$

- Since it is an easy principle, it is applied in advanced system dynamic analysis.
- This method is suitable to control the response characteristics of the structure in time domain.

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④ Rayleigh's method :

- It is an extension of Energy method

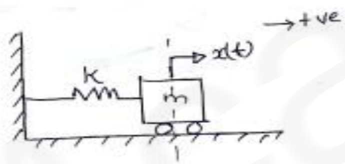
- If K.E is max → P.E will be min (or) zero
  If P.E is max → K.E will be min (or) zero.

$$\boxed{(P.E)_{max} = (K.E)_{max}} = (T.E)_{system}$$

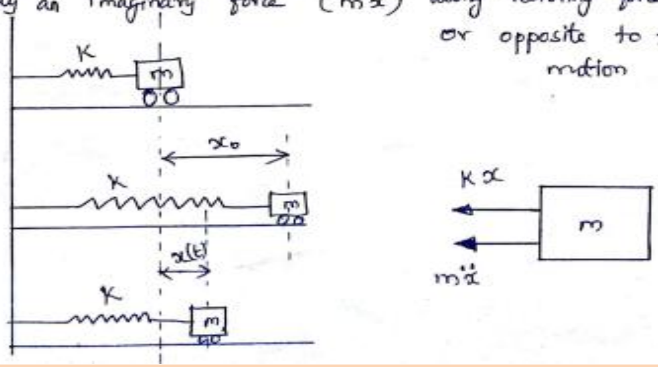
⑤ D'Alembert's principle :

- He converted the dynamic problem into equivalent static problem

- ∴ Imaginary concept



- Initially spring-mass system is at rest
- Apply F(t) → spring mass system displaced by x_0
- Release this force - mass will vibrate
- Apply an imaginary force ($m\ddot{x}$) along restoring force direction or opposite to the direction of motion



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$\Sigma H = 0$

$-Kx - m\ddot{x} = 0$

$$\boxed{m\ddot{x} + Kx = 0}$$

- D'Alembert's principle is a concept, it cannot be applied to reality because at any instant of time $m\ddot{x}$ is keep on varying
- Program controller is required to make it static
- This concept is used in controll Algorithm (Intelligent structure)
- We cannot able to apply this concept for multi degree of freedom system.

undamped free vibration $m\ddot{x} + kx = 0$

$$\ddot{x} + \left(\frac{k}{m}\right)x = 0$$

$$\omega^2 = \frac{k}{m}$$

Natural frequency $\omega_n = \sqrt{\frac{K}{m}}$ $\frac{rad}{s}$

$\omega_n \rightarrow$ circular frequency (or) angular velocity

$$f = \text{cyclic frequency} = \frac{\omega_n}{2\pi}$$

Time period $T = \frac{1}{f}$

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Simple Harmonic motion : (SHM)

In mechanics & physics, SHM is a type of periodic motion (or) oscillation, where the restoring force is directly proportional to the displacement and acts in the opposite direction to that of displacement from its equilibrium position. It can be represented by sine or cosine function.

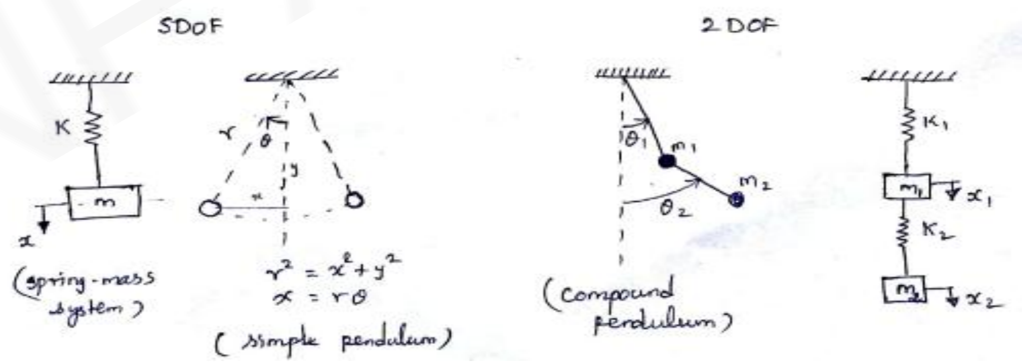
Ex: Simple pendulum.

Degree of freedom: -

The minimum number of independent coordinates required to specify the position of a system completely at any instant.

Modal Analysis :-

A modal Analysis determines the vibration characteristics (Natural frequency & mode shapes) of a structure or machine component.



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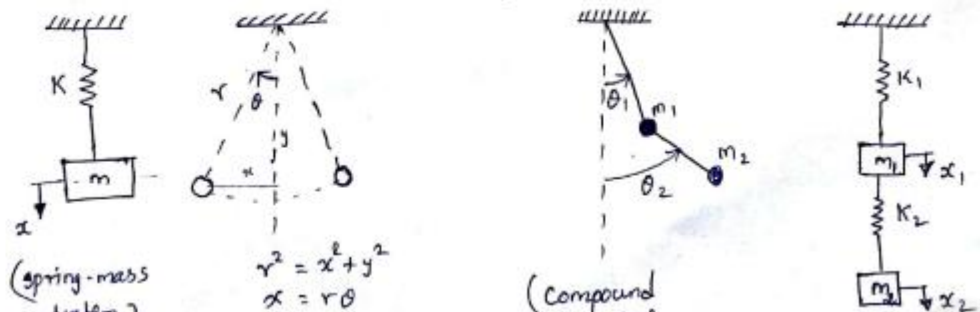
SDOF 2 DOF



(spring-mass system) (simple pendulum) (Compound pendulum)

One parameter is enough to specify the position of mass

Two parameters required x_1, x_2 (θ_1, θ_2)

Resonance:

When the frequency of external excitation is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This is called resonance.

Mechanical systems: (or) **Parts of vibrating system:**

(i) Spring
(ii) Mass
(iii) Damper

Beats:

When two simple Harmonic motions of slightly different frequencies are added, the resulting motion is known as beat.

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Types of vibration:

(i) Free vibration:

After disturbing the system the external excitation is removed, then the system vibrates on its own. This type of vibration is called free vibration.

Ex: Simple pendulum

(ii) Forced vibration:

The vibration which is under the influence of external force is called forced vibration.

Ex: Electric bells.

Linear vibration:

If in a vibratory system mass, spring, and damper behave in a linear manner, the vibrations caused are known as linear vibration.

Non-Linear vibration:

If any of the basic components of a vibratory system behaves non linearly, the vibration is called non-linear. Linear vibration becomes non-linear for very large amplitude of vibration.

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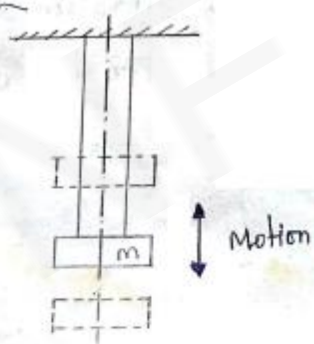
Damped vibration:

If the vibratory system has a damper, the motion of the system will be opposed by it and the energy of the system will be dissipated in friction. This type of vibration is called damped vibration.

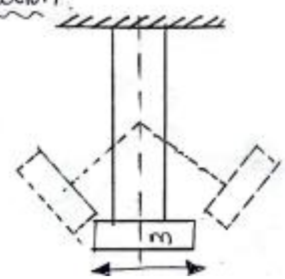
Undamped vibration:

The system having no damper is known as undamped vibration.

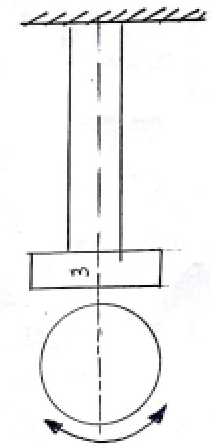
Longitudinal vibration:



Transverse vibration:



Torsional vibration:



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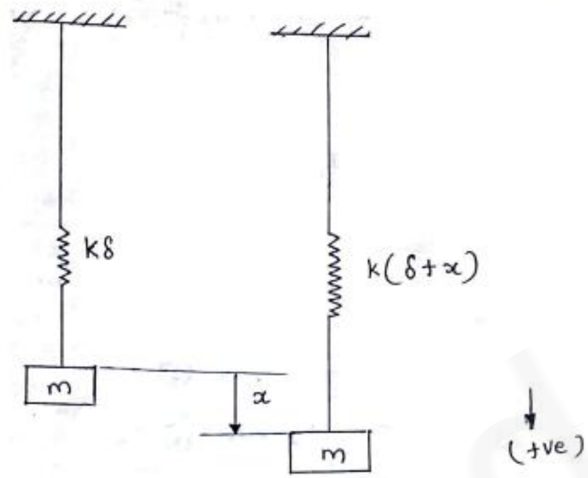
Equation of spring mass system in vertical position



$\delta_{st} \rightarrow$ static deflection of spring

$W = k\delta_{st}$

Newton's method:

(+ve) Inertia force = Restoring force

$$m\ddot{x} = W - k(\delta_{st} + x)$$

$$= k\cancel{\delta_{st}} - k\cancel{\delta_{st}} - kx \qquad W = k\delta_{st}$$

$$\boxed{m\ddot{x} + kx = 0}$$

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Roots

$$P = \pm i\sqrt{\frac{k}{m}} = \pm i\omega_n$$

$$x(t) = C.F = A_1 \cos\omega_n t + A_2 \sin\omega_n t \longrightarrow (1)$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad \frac{rad}{s}$$

frequency of vibration $f_n = \dfrac{\omega_n}{2\pi}$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad Hz$$

B.C

@ $t=0 \Rightarrow x = x_0 \longrightarrow (2)$

@ $t=0 \Rightarrow \dot{x} = \dot{x}_0 \longrightarrow (3)$

$$\dot{x}(t) = -A_1\omega_n \sin\omega_n t + A_2\omega_n \cos\omega_n t \longrightarrow (4)$$

sub (2) in (1)

$$x_0 = A_1 \Rightarrow \boxed{A_1 = x_0}$$

sub (3) in (4)

$$\dot{x}_0 = A_2\omega_n \Rightarrow \boxed{A_2 = \frac{\dot{x}_0}{\omega_n}}$$

$$\boxed{x(t) = x_0 \cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t}$$

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Problem -

Determine the displacement, velocity & acceleration of the mass of a SMS with $k = 500 \frac{N}{m}$, $m = 2\,kg$, $x_0 = 0.1\,m$, $\dot{x}_0 = 5 \frac{m}{s}$

Given :-

$k = 500 \frac{N}{m}$

$m = 2\,kg$

$x_0 = 0.1\,m$

$\dot{x}_0 = 5 \frac{m}{s}$

To find :-

$x, \dot{x}, \ddot{x} = ?$

Soln :-

$x = A_1 \cos\omega_n t + A_2 \sin\omega_n t$

$\omega_n = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{500}{2}}$

$$\boxed{\omega_n = 15.811 \frac{rad}{s}}$$

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$$A_1 = x_0 = 0.1 m$$

$$A_2 = \frac{\dot{x}_0}{\omega_n} = \frac{5}{15.811} = 0.32 m$$

$$x = 0.1 \cos 15.811t + 0.32 \sin 15.811t$$

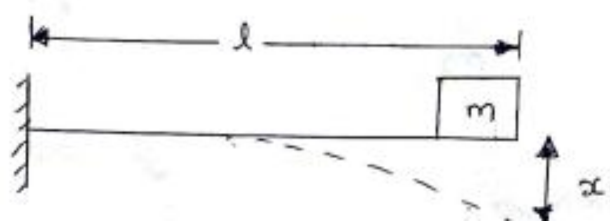
$$\dot{x} = -(0.1)(15.811) \sin 15.811t + (0.32)(15.811) \cos 15.811t$$

$$\dot{x} = 5.059 \cos 15.811t - 1.581 \sin 15.811t$$

$$\ddot{x} = -[79.98 \sin 15.811t + 24.99 \cos 15.811t]$$

Problem:

Determine the natural frequency of the mass m placed at one end of a cantilever beam of negligible mass as shown in fig.



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Soln :

$$\text{Deflection} = \frac{Wl^3}{3EI}$$

$$\text{Stiffness } (k) \text{ of beam} = \frac{\text{Load}}{\text{deflection}}$$

$$= \frac{\cancel{W}}{\cancel{W}l^3/3EI}$$

$$k = \frac{3EI}{l^3}$$

EI – flexural rigidity of the beam

Eqn of motion

$$m\ddot{x} + kx = 0$$

$$m\ddot{x} + \frac{3EI}{l^3}x = 0 \Rightarrow \ddot{x} + \frac{3EI}{ml^3}x = 0$$

$$\omega_n = \sqrt{\frac{3EI}{ml^3}} \quad \frac{rad}{s}$$

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{ml^3}} \quad Hz}$$

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Problem:

An automobile have the natural frequency of 20 rad/s without passengers & 17.32 $\frac{rad}{s}$ with passenger mass 500kg. Find the mass & stiffness of the automobile by treating it as single DOF.

soln

without Passenger:

$$\omega_n = \sqrt{\frac{k}{m_1}}$$

$$20^2 = \frac{k}{m_1}$$

$$\boxed{k = 20^2 m_1}$$

with passenger:

$$\omega_n = \sqrt{\frac{k}{m_1 + 500}}$$

$$(17.32)^2 = \frac{k}{m_1 + 500}$$

$$k = (17.32)^2 (m_1 + 500)$$

$$20^2 m_1 = (17.32)^2 m_1 + (17.32)^2 (500)$$

$$\boxed{m_1 = 1499.64 \text{ kg}}$$

$$k = 599.856 \times 10^3 \frac{N}{m}$$

$$\boxed{k = 599.86 \frac{N}{mm}}$$

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